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# INFORMATION CAPACITY OF ADDITIVE GAUSSIAN CHANNELS WITH JAMMING

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Information capacity is determined for the matched Gaussian channel when jamming is added to the ambient noise. The problem is modeled as a zero-sum two-person game, with mutual information as the payoff function. The saddle value, a saddle point, and an optimum jammer strategy are given for both the finite-dimensional and the infinite-dimensional channel.


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## Summary

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## Introduction

A game theoretic approach to information capacity of a channel subject to jamming dates back to (at least) Blachman's 1957 paper [1]. Apparently, no significant work was done on this problem until the 1980's. In recent years, some work has been accomplished. McEliece and Stark [2] gave a treatment of the one-dimensional problem. Conference papers have also been presented [3], [4].

Some of the previous work assumes that the jammer has control over all the significant noise in the channel. In practice, this is not always the case; the most challenging (to the coder) situations arise when the signal-to-ambient-noise ratio is already low. The jammer's objective should be optimal use of his available energy in combination with the ambient noise. The actual channel in this case has the output

$$Y = X + W + J$$

where $W$ is the ambient Gaussian noise, $X$ the transmitted signal, and $J$ the jamming noise. These processes are described by probability measures $\mu_Y$, $\mu_X$, $\mu_W$, and $\mu_J$, defined on an appropriate space containing the sample functions. The mutual information of interest is $I(X,Y) = I(\mu_{XY})$, where $\mu_{XY}(C) = \mu_X \otimes \mu_W \otimes \mu_J\{(x,w,v)\colon (x, x+w+v) \in C\}$, and $\otimes$ denotes product measure. Permitting

the jammer to have control on only part of the interference substantially changes both the nature of the problem and the solution.

We will summarize recent results on this problem under the following assumptions. First, all the processes are zero mean with sample paths that belong to a real separable Hilbert space, H, with inner product $\langle\cdot,\cdot\rangle$ and associated norm $\|\cdot\|$. It will be assumed, WLOG, that the support of the W probability $\mu_W$ is all of H; equivalently, that the range of the W covariance operator, $R_W$, is dense in H. The class of admissible signal processes X consists of all those such that $\mu_X[\text{range}(R_W^{1/2})] = 1$ and $E_{\mu_X}\|R_W^{-1/2}x\|^2 \leq P$. The jammer's input to the channel is described by the probility $\mu_J$, and is required to satisfy $E_{\mu_J}\|x\|^2 \leq P$. Jamming noise, ambient noise, and signal are mutually independent.

By a result of Ihara [5], the capacity of this channel (for a fixed jammer covariance $R_J$) is minimized when J + W is Gaussian. Thus, the jammer should always choose Gaussian jamming, and this assumption is made throughout. From the coder's viewpoint, a Gaussian jamming signal produces a mismatched Gaussian channel. Capacity of such channels was determined in [6]; the application to jamming channels was a principal motivation for that work.

We are assuming here that the channel noise probability, $\mu_W$, is countably additive; equivalently, that $R_W$ is trace-class. Thus, $R_W = \Sigma_{n\geq 1}\lambda_n e_n \otimes e_n$, where $\lambda_n > 0$ for $n \geq 1$, $(\lambda_n)$ is non-increasing, $\Sigma_{n\geq 1}\lambda_n < \infty$, and $(e_n \otimes e_n)u \equiv \langle e_n,u\rangle e_n$ for u in H. The jammer's constraint requires that the jamming covariance be trace-class. Moreover, from the results of [6], the jammer's signal must be such that $R_J = R_W^{1/2} S R_W^{1/2}$, where S is a self-adjoint operator whose domain $\mathfrak{D}(S)$ contains $\text{range}(R_W^{1/2})$, and is such that $(I+S)^{-1}$ exists and is bounded. In the present case, $(I+S)^{-1}$ necessarily exists and is bounded, since S is

non-negative. Since the constraint on the jammer is of the form $E_{\mu_J}\|x\|^2 \leq P_2$, one has the equivalent constraint Trace $R_J$ = Trace $R_W^{1/2} S R_W^{1/2} \leq P_2$.

Thus, we have the jammer constraint,

$$\sum_{n\geq 1} \langle R_W^{1/2} S R_W^{1/2} e_n, e_n \rangle = \sum_{n\geq 1} \lambda_n \langle S e_n, e_n \rangle \leq P_2,$$

and the coder's constraint

$$\text{Trace } R_W^{-1/2} R_X R_W^{-1/2} = \sum_{n\geq 1} \langle R_X e_n, e_n \rangle / \lambda_n \leq P_1.$$

The jammer's strategy lies in the choice of the operator S. The coder's strategy lies in the choice of the operator $R_X$. A partial characterization of the optimum S is given by the following key result.

<u>Prop. 1</u>. *The jammer's minimax strategy can be achieved by taking* $S = \Sigma_{i\geq 1} \gamma_i e_i \otimes e_i$, *where* $\Sigma_{i\geq 1} \lambda_i \gamma_i \leq P_2$, $\gamma_i \geq 0$ *for* $i \geq 1$, *with* $(\gamma_i)$ *non-decreasing.*

In the following two sections, we summarize our results for the finite-dimensional channel and the infinite-dimensional channel.

It should perhaps be emphasized that we are limiting consideration to the game-theoretic problem where mutual information is used as the payoff function.

## <u>Finite-Dimensional Channel</u>

Here we suppose that all sample paths are in $\mathbb{R}^M$. If the jammer has selected the strategy given by $S = \Sigma_{i=1}^M \gamma_i e_i \otimes e_i$, $\gamma_1 \leq \gamma_2 \leq \ldots \leq \gamma_M$, then it is well-known that the capacity of the channel is given by

$$C_W^M(P) = \tfrac{1}{2}\sum_{i=1}^{K}\log\left[\frac{P_1 + \sum_{j=1}^{K}\gamma_j + K}{K(1+\gamma_i)}\right]$$

where K is the largest integer such that $P_1 + \sum_{i=1}^{K}\gamma_i \geq K\gamma_K$, and the coder's optimum strategy is to select his covariance matrix $R_X$ to be given by

$$R_X = \sum_{n=1}^{M}\tau_n\left[(R_W+R_J)^{1/2}e_n \otimes (R_W+R_J)^{1/2}e_n\right],$$

where $\tau_n = (\sum_{i=1}^{K}\beta_i + P - K\beta_n)(1+\beta_n)^{-1}K^{-1}$ for $n \leq K$

$= 0$ $\qquad n > K.$

The coder's strategy can be rewritten as

$$z_i = \frac{P_1 + \sum_{j=1}^{K}\gamma_j}{K} - \gamma_i \qquad i \leq K$$

$$= 0 \qquad i > K$$

where $R_X = \sum_{i=1}^{K} z_i(1+\gamma_i)[(R_W+R_J)^{1/2}e_i \otimes (R_W+R_J)^{1/2}e_i]$.

Inserting this into the preceding expression for the capacity, one obtains

$$C_W^M(P) = \tfrac{1}{2}\sum_{i=1}^{M}\log\left[1 + z_i(1+\gamma_i)^{-1}\right] \equiv F(z,\gamma)$$

where $z \in U$, $\gamma \in V$,

$$U = \{z \text{ in } \mathbb{R}^M;\ z_i \geq 0 \text{ for } i \leq M,\ \sum_{i=1}^{M} z_i \leq P_1\}$$

$$V = \{\gamma \text{ in } \mathbb{R}^M;\ \gamma_i \geq 0 \text{ for } i \leq M,\ \sum_{i=1}^{M}\lambda_i\gamma_i \leq P_2\}.$$

In this form, F is a real-valued function on U×V, where U and V are bounded, closed, and convex. $F(x,\cdot)$ is continuous and strictly concave on V (every x

in U), while $F(\cdot,y)$ is continuous and strictly convex on U (every y in V). Thus, by the von Neumann minimax theorem [7], the zero-sum two-person game with F as payoff function has a unique saddle point: a point $(z^*,\tau^*)$ such that

$$\max_{z\in U}\min_{\tau\in V} F(z,\tau) = \min_{\tau\in V}\max_{z\in U} F(z,\tau) = F(z^*,\tau^*).$$

Using the above results for the coder's optimum strategy when S (equivalently, $\tau$) is given, one can write $F(z,\tau)$ as a function only of $\tau$, in the form

$$F_0(\tau) = \tfrac{1}{2}\sum_{i=1}^{M} \log\left[\frac{P_1 + \sum_{j=1}^{M}\tau_j + M}{M(1+\tau_i)}\right].$$

This function, $F_0$, is the expression that the jammer must seek to minimize. From the above, minimization over V of this function by $\tau = \tau^*$ will give the jammer's minimax strategy and the coder's maximin strategy.

This problem can be solved, after some rearrangement, by constrained minimization. Thus, define, for $0 \le K \le M$,

$$\Lambda_K = \{(z,\tau)\colon 0 = \tau_1 = \tau_2 = \dots = \tau_K < \tau_{K+1} \le \tau_{K+2} \le \dots \le \tau_M,$$
$$\sum_{i=1}^{M}\lambda_i\tau_i \le P_2,\quad z_1 \ge z_2 \ge \dots \ge z_M \ge 0,\ \sum_{j=1}^{M} z_i \le P_1\}.$$

The objective function $F_0$ is strictly convex, with convex constraint set. Thus, a unique minimum exists. The sets $\Lambda_i$, $0 \le i \le M-1$, are disjoint. The procedure is to sequentially search these sets, beginning with $\Lambda_0$, until a solution to the minimization problem is obtained. The solution is given by the following theorem.

*<u>Theorem 1</u>. Let K be the smallest integer such that*

$$P_2 > \sum_{n=K+1}^{M} \frac{(\lambda_{K+1}-\lambda_n)[(M-K)(P_1+K) - Ky_K^M]\lambda_n/M}{P_2+\Sigma_{i=K+1}^{M}\lambda_i+[(M-K)(P_1+K-Ky_K^M]\lambda_n/M} .$$

*Then the saddle point* $(z^*,\gamma^*)$ *is given by*

$$\gamma_i^* = 0 \qquad\qquad i \leq K$$

*and for* $i > K$,

$$\gamma_i^* = \frac{[P_2+\Sigma_{j=K+1}^{M}\lambda_j](K+P_1+y_K^M)}{M(P_2+\Sigma_{k=K+1}^{M}\lambda_k) + [(M-K)(P_1+K) - Ky_K^M]\lambda_i} - 1$$

$$z_1^* = \ldots = z_K^* = \frac{P_1 + \Sigma_{i=K+1}^{M}\gamma_i^*}{M}$$

$$z_i^* = z_1^* - \gamma_i^* \qquad\qquad i > K$$

*where* $y_K^M$ *is defined by*

$$\left[\frac{K+P_1+y_K^M}{M}\right]^{-1} = \sum_{n=K+1}^{M} \frac{\lambda_n}{P_2+\Sigma_{i=K+1}^{M}\lambda_i+[(M-K)(P_1+K)-Ky_K^M]\lambda_n/M} .$$

*This gives the saddle value*

$$F(z^*,\gamma^*) = \frac{M}{2}\, log(1+z_1^*) - \tfrac{1}{2} \sum_{i=K+1}^{M} log(1+\gamma_i^*).$$

<u>Remark</u>: It is known (see, e.g., [6]) that the capacity of this channel without jamming is $\frac{M}{2}\log(1 + P_1/M)$. For a sense of the degradation that can be caused by an intelligent jammer, suppose that the saddle point lies in $\Lambda_0$. Then the saddle value, or the capacity when the jammer chooses his minimax strategy, is

$$F(z^*,\tau^*) = \frac{M}{2}\log(1+\frac{P_1}{M}) - \frac{1}{2}\sum_{i=1}^{M}\log\left[\frac{(P_2+TrR_W)(M+P_1)}{(P_2+TrR_W+P_1\lambda_i)M}\right]$$

$$= \frac{M}{2}\log(1+\frac{P_1}{M}) - \frac{1}{2}\sum_{i=1}^{M}\log\left[\frac{M+P_1}{\left[1+\frac{P_1\lambda_i}{P_2+TrR_W}\right]M}\right].$$

An immediate consequence of Theorem 1 follows.

*Corollary 1: Suppose that one uses the constraint* $E_{\mu_J}\|x\|_W^2 \le P_2$ *for the jammer. The saddle point solution is then given by Theorem 1, setting* $\lambda_1 = \lambda_2 = \ldots = \lambda_M = 1$. *The saddle point solution* $(z^*,\tau^*)$ *is contained in* $\Lambda_0$, *and has the form:*

$$\tau_i^* = \frac{P_2}{M} \qquad i = 1,2,\ldots,M$$

$$z_i^* = \frac{P_1+P_2}{M} - \frac{P_2}{M} = \frac{P_1}{M} \qquad i = 1,2,\ldots,M.$$

*Thus, the saddle value of F is*

$$F(z^*,\tau^*) = \frac{M}{2}\log\left[\frac{P_2+P_1+M}{P_2+M}\right] = \frac{M}{2}\log\left[1+\frac{P_1}{P_2+M}\right].$$

## Infinite-Dimensional Channel

For the infinite-dimensional channel, let $\mathbb{R}_+^\infty$ be the set of all real-valued sequences $x = (x_1,x_2,\ldots.)$ such that $x_i \ge 0$ for $i \ge 1$. The admissible strategies for the coder and for the jammer are then defined by U and V, where

$$U = \{z \text{ in } \mathbb{R}_+^\infty: \sum_{n\ge1} z_n \le P_1\}$$

$$V = \{\tau \text{ in } \mathbb{R}_+^\infty: \sum_{n\ge1} \lambda_n\tau_n \le P_2\}.$$

We obtain a solution by showing that for a specific choice of $(z^*,\gamma^*)$, inferred in part from the finite-dimensional result,

$$\sup_{z\in U}\inf_{\gamma\in V} F(z,\gamma) \geq F(z^*,\gamma^*) \geq \inf_{\gamma\in V}\sup_{z\in U} F(z,\gamma).$$

Since it always holds that $\sup_U \inf_V \leq \inf_V \sup_U F$, this shows that $(z^*,\gamma^*)$ is a saddle point, with the definition as in the following theorem.

<u>Theorem 2</u>. *A saddle point $(z^*,\gamma^*)$ is given as follows.*

$$\gamma_n^* = 0 \qquad n \leq K$$

$$1+\gamma_n^* = \frac{P_2 + \Sigma_{i=K+1}^{\infty}\lambda_i}{P_2 + \Sigma_{j=K+1}^{\infty}\lambda_j + (P_1-K\theta)\lambda_n}(1+\theta) \qquad n > K$$

$$z_n^* = \theta \qquad n \leq K$$

$$z_n^* = \theta - \gamma_n^* \qquad n > K.$$

*K is the smallest integer $\geq 0$ such that*

$$P_2 > \frac{(P_1-K\theta)\lambda_{K+1} - \theta\Sigma_{i=K+1}^{\infty}\lambda_i}{\theta}.$$

*$\theta$ is defined by*

$$(1+\theta)^{-1} = \sum_{n=K+1}^{\infty} \frac{\lambda_n}{P_2 + \Sigma_{j=K+1}^{\infty}\lambda_j + (P_1-K\theta)\lambda_n}.$$

*The saddle value is then given by*

$$F(z^*,\gamma^*) = \frac{K}{2}\log(1+\theta) + \tfrac{1}{2}\sum_{n=K+1}^{\infty}\log\left[1 + \frac{(P_1-K\theta)\lambda_n}{P_2+\Sigma_{i=K+1}^{\infty}\lambda_i}\right].$$

<u>Remark</u>. If $P_2 = 0$ (no jamming), then the capacity is $\frac{P_1}{2} =$ $\lim_{K\to\infty} \frac{K+1}{2} \log\left[1 + \frac{P_1}{K+1}\right]$. If $P_2 > 0$, then for the value of K giving the saddle value in the above theorem, the saddle value is $\leq \frac{K+1}{2} \log\left[1 + \frac{P_1}{K+1}\right]$. The jammer wishes to choose K as small as possible.

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